Das Skalarprodukt von Vektoren, die Größe von Winkeln - Lösung

1. $\vec{a} \circ \vec{b} = \begin{pmatrix} 2 \\ 14 \\ -5 \end{pmatrix} \circ \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 2 \cdot 5 + 14 \cdot 0 + (-5) \cdot 2 = 0;$

   $\vec{a} \circ \vec{c} = \begin{pmatrix} 2 \\ 14 \\ -5 \end{pmatrix} \circ \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} = 0; \; \vec{b} \circ \vec{c} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} \circ \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} = -6;$

2. $\vec{a} \circ \vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \circ \begin{pmatrix} 3 \\ 2k \\ -k \end{pmatrix} = 4 \cdot 3 + 5 \cdot 2k + 6 \cdot (-k) =$

   $12 + 10k - 6k = 12 + 4k = 0; \Rightarrow k = -3.$

3. $\vec{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$ $\quad \vec{a} \circ \vec{c} = 0: -4c_1 + 1c_2 + 3c_3 = 0.$

   $\vec{b} \circ \vec{c} = 0: -2c_1 + 4c_2 - 9c_3 = 0.$

   Wähle z. B. $c_1 = 1$:

   (I) $1c_2 + 3c_3 = 4 \quad | \cdot 3$

   (II) $4c_2 - 9c_3 = 2$

   (I‘) $3c_2 + 9c_3 = 12$

   (I‘) +(II) $7c_2 = 14; \Rightarrow c_2 = 2;$

   In (I) $2 + 3c_3 = 4; \; 3c_3 = 2; \; c_3 = \frac{2}{3};$

   $\vec{c} = \begin{pmatrix} 1 \\ 2 \\ \frac{2}{3} \end{pmatrix}$ oder: $\vec{c} = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} -0{,}5 \\ -1 \\ -\frac{1}{3} \end{pmatrix}.$

4. $\overrightarrow{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}; \; \overrightarrow{BC} = \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}; \overrightarrow{CD} = \begin{pmatrix} -0{,}5 \\ -2 \\ -2 \end{pmatrix}; \overrightarrow{DA} = \begin{pmatrix} -4{,}5 \\ -3 \\ 3 \end{pmatrix}$

   a) $\overrightarrow{AB} = -2 \cdot \overrightarrow{CD}; \Rightarrow [AB] \, || [CD] \Rightarrow$ Trapez.

   b) $\overline{AB} = \sqrt{1^2 + 2^2 + 2^2} = 3; \overline{BC} = \sqrt{4^2 + 2^2 + (-4)^2} = 6;$

   $\overline{CD} = \sqrt{(-0{,}5)^2 + 1^2 + 1^2} = 1{,}5;$

   $\overline{DA} = \sqrt{(-4{,}5)^2 + (-3)^2 + 3^2} = \sqrt{38{,}25} = 1{,}5\sqrt{17};$

   c) $\cos\alpha = \frac{\overrightarrow{AB} \circ \overrightarrow{AD}}{\overline{AB} \cdot \overline{AD}} = \frac{1 \cdot 4{,}5 + 2 \cdot 3 + 2 \cdot (-3)}{3 \cdot \sqrt{38{,}25}} = \frac{4{,}5}{3 \cdot \sqrt{38{,}25}} = \frac{1}{\sqrt{17}}; \Rightarrow \alpha \approx 76{,}0°$

   d) $\cos\beta = \frac{\overrightarrow{BA} \circ \overrightarrow{BC}}{\overline{BA} \cdot \overline{BC}} = \frac{-1 \cdot 4 + (-2) \cdot 2 + (-2) \cdot (-4)}{3 \cdot 6} = \frac{0}{18} = 0; \Rightarrow \beta = 90°;$

   Trapez $\beta = \gamma = 90°; \; \delta = 180° - \alpha \approx 104{,}0°$